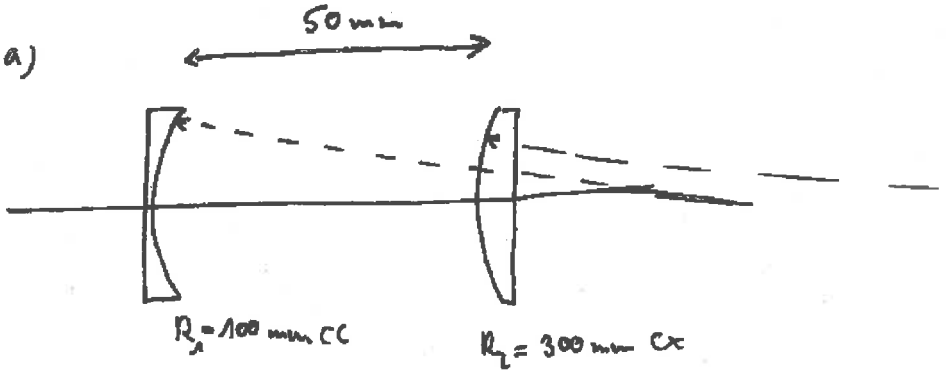
Sheet 5

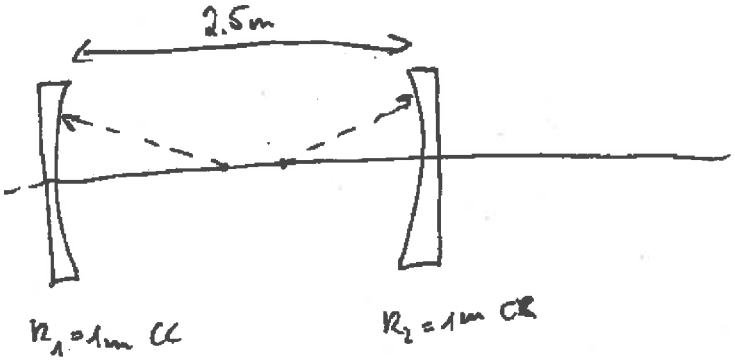
P13

a)



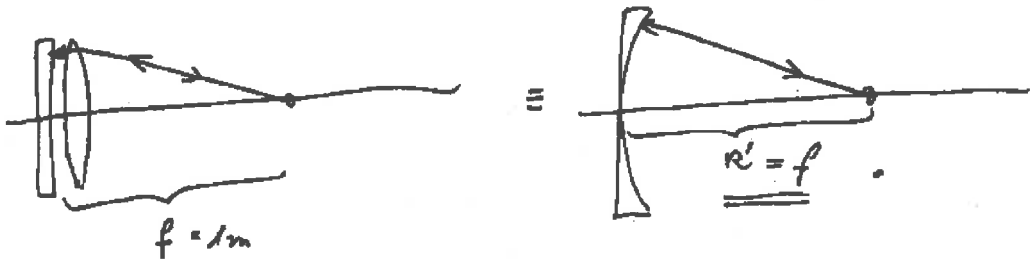
partial radii overlap ⇒ stable resonator

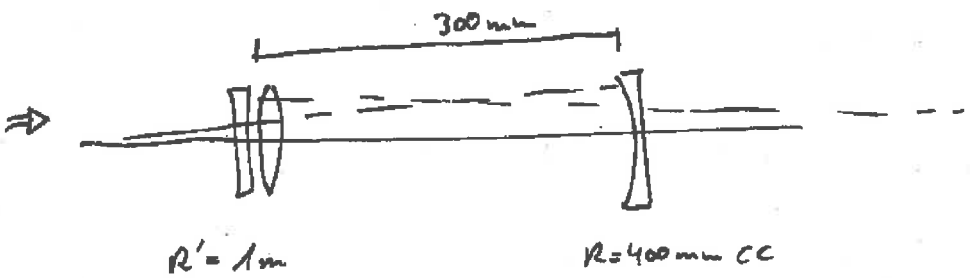
b)



no radii overlap ⇒ unstable resonator

c)

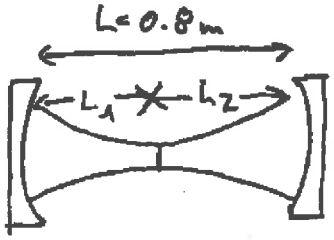




partial overlap ⇒ stable resonator

P13

b)



$R_1 = 1\,m$ $\quad$ $R_2 = 3\,m$

$$g_1 = 1 - \frac{L}{R_1} \qquad g_2 = 1 - \frac{L}{R_2} \Rightarrow \quad g_1 = 0.2$$

$$g_2 = 0.73$$

$$L_1 = \frac{g_2(1-g_1)}{g_1 + g_2 - 2g_1g_2} \qquad L = 0.73\,m$$

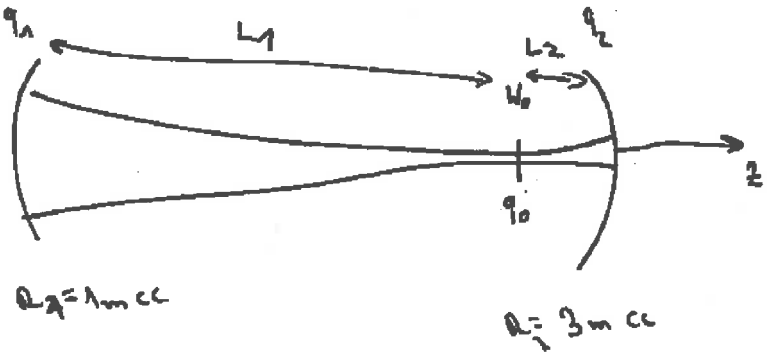
$$L_2 = \frac{g_1(1-g_2)}{g_1 + g_2 - 2g_1g_2} \qquad L = 0.067\,m$$

$$z_R^2 = \frac{g_1g_2(1-g_1g_2)}{(g_1+g_2-2g_1g_2)^2} \cdot L^2 \Rightarrow \quad z_R = 0.442\,m$$

$$z_R = \frac{\pi w_0^2}{\lambda} \Rightarrow \quad w_0 = \sqrt{\frac{z_R \lambda}{\pi}} = 298\,\mu m$$

$$\Rightarrow \frac{1}{e^2}\ \text{diameter} = 2w_0 = 596\,\mu m$$

A13 5)



$$q_0 = i z_R = i \cdot 0.442\,m$$

$$q_1 = q_0 - L_1 = -0.73\,m + i \cdot 0.442\,m$$

$$q_2 = q_0 + L_2 = 0.067\,m + i \cdot 0.442\,m$$

$$\frac{1}{q_1} = \frac{1}{R_1} - i \frac{\lambda}{\pi w^2}$$

$$\frac{1}{q_1} = \frac{1}{-0.73\,m + i \cdot 0.442\,m} = \frac{-0.73\,m - i \cdot 0.442\,m}{(0.73\,m)^2 + (0.442\,m)^2} = -1.002\,m^{-1} - i \cdot 0.607\,m^{-1}$$

$$\Rightarrow 2w_1 = 2\sqrt{\frac{\lambda}{\pi \cdot \mathrm{Im}\left(\frac{1}{q_1}\right)}} = \underline{\underline{1.15\,mm}}$$

$$\frac{1}{q_2} = \frac{1}{0.067\,m + i \cdot 0.442\,m} = \frac{0.067\,m - i \cdot 0.442\,m}{(0.067\,m)^2 + (0.442\,m)^2} = 0.335\,m^{-1} - i \cdot 2.212\,m^{-1}$$

$$\Rightarrow 2w_2 = 2\sqrt{\frac{\lambda}{\pi\, \mathrm{Im}\left(\frac{1}{q_2}\right)}} = \underline{\underline{603\,\mu m}}$$

P14

a) $$F = \frac{a^2}{\lambda L} = \frac{(12.6\,\text{mm}/2)^2}{1064\,\text{nm} \cdot 300\,\text{mm}} = \underline{\underline{124,3}}$$

b) $$\delta_\infty = 5 \cdot 10^{-4} \cdot F^{-7.67} = 4.79 \cdot 10^{-20}$$

$$\delta_{10} = 10^{-2} \cdot F^{-7.67} = 8.6 \cdot 10^{-19}$$

c) $$\delta\nu = -\frac{c}{2\pi L} \ln(0.998^2) = 637\,\text{kHz}$$

$$\Delta\nu = \frac{c}{2\pi L} = 500\,\text{MHz}$$

$$\Rightarrow F_c = \frac{\Delta\nu}{\delta\nu} = \underline{\underline{785}}$$

P15

a) $L_0^\ell = \frac{1}{0!} x^{-\ell} e^x \cdot x^\ell e^{-x} = \underline{\underline{1}}$

$$L_1^\ell = \frac{1}{1!} x^{-\ell} e^x \frac{d}{dx}\left(x^{1+\ell} e^{-x}\right)$$

$$= x^{-\ell} e^x \left[(1+\ell) x^\ell e^{-x} + x^{1+\ell}(-1) e^{-x}\right]$$

$$= \underline{\underline{1+\ell-x}}$$

$$L_2^\ell = \frac{1}{2!} x^{-\ell} e^x \frac{d^2}{dx^2}\left(x^{2+\ell} e^{-x}\right)$$

$$= \frac{1}{2} x^{-\ell} e^x \frac{d}{dx}\left[(2+\ell) x^{1+\ell} e^{-x} + x^{2+\ell}(-1) e^{-x}\right]$$

$$= \frac{1}{2} x^{-\ell} e^x \left[(2+\ell)(1+\ell) x^\ell e^{-x} + (2+\ell) x^{1+\ell}(-1) e^{-x}\right.$$
$$\left. + (2+\ell) x^{1+\ell}(-1) e^{-x} + x^{2+\ell}(-1)(-1) e^{-x}\right]$$

$$= \underline{\underline{\frac{1}{2}(2+\ell)(1+\ell) - (2+\ell)\cdot x + \frac{1}{2}x^2}}$$

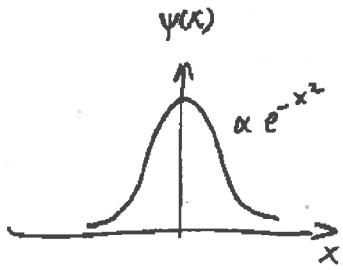
P15

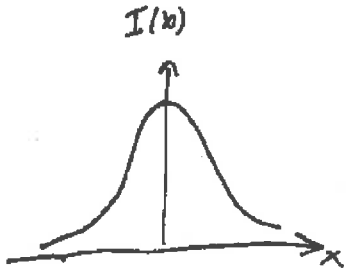
b) $\psi(x) \propto x^l L_p^l(4x^2)\, e^{-x^2}$

$I(x) \propto |\psi(x)|^2 \propto x^{2l} \left[L_p^l(4x^2)\right]^2 e^{-2x^2}$

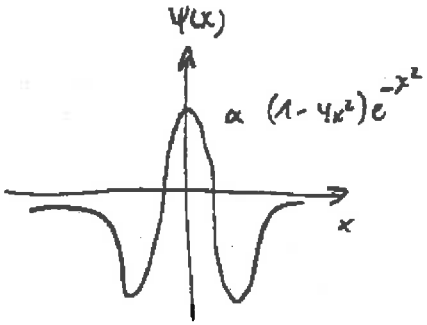
$\Rightarrow \quad \psi(x) \propto L_p^0(4x^2)\, e^{-x^2}$

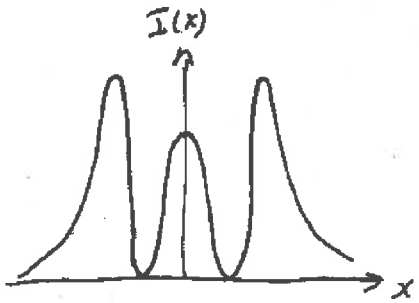
TEM_{00}





TEM_{01}





TEM_{02}